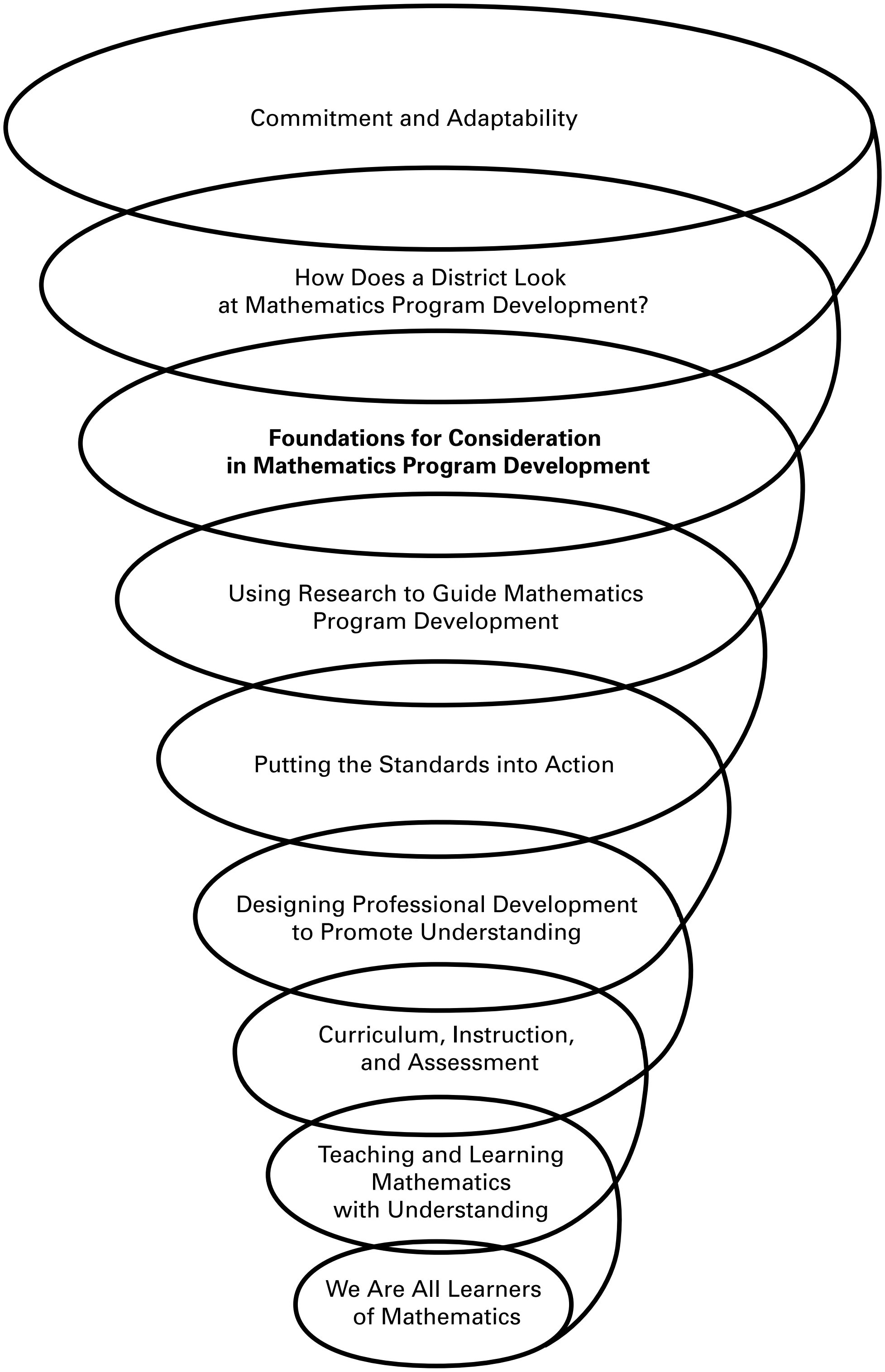
Planning Curriculum in



Mathematics

Spring (spring) *n.* 1. a source; origin; or beginning: *a spring into action.* 2. elasticity; resilience: *a spring forward with commitment.* (Morris, W., ed., 1971. *The American Heritage Dictionary of the English Language.* New York: American Heritage Publishing Company, Inc. and Houghton Mifflin Company, 1250, 2nd definition.)

Foundations for Consideration in Mathematics Program Development

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When the agency decided, in response to school district requests, to produce guides for planning curriculum certain principles were identified that transcended discipline lines and served as foundations for the task force efforts. Among them were: teaching and learning with understanding; and curriculum, instruction, assessment, and professional development and the interrelationships between and among them (in previous chapters). In addition, tenets upon which agency efforts rested were identified: equity, enlightened use of technology, and a specific agency thrust, service learning (all discussed in this chapter). The mathematics guide task force felt that foundations of pertinence to mathematics at this time were: openness and time (in this chapter), and commitment and adaptability (final chapter in this document). Collectively, these foundations undergrid all of the endeavors undertaken in planning curriculum; indeed, they often weigh heavily in the success or failure of viable efforts.

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Openness

> Openness is a term that has gained acceptance to designate a willingness to learn, a desire to capitalize on the findings of research, and a genuine concern for improvement. So often in the preparation of these materials, previous efforts of the department, of national and state groups, and of researchers to effect improvements similar to those being called for in the guide, surfaced. Yet there still seems to be, in some circles, uncertainty, ambivalence, and lack of conviction regarding what needs to be done mathematically as we go forward into a new millennium. With mushrooming knowledge, instantaneous communication, and hard evidence, it would seem that improvement would come more readily. It is openness to the realm of possibilities that will make this happen.

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Our world has become smaller because of instant communication. Knowledge acquisition has become easier because we can access information online, download it, and instantly put it to use. We have literally at our fingertips

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—Fennema, Sowder, and Carpenter 1999, 185

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research on how mathematics is best taught and learned (Schoenfeld, Studies in Mathematical Thinking and Learning Series), on what kind of mathematics is worthy of understanding (Romberg and Kaput 1999), on mathematical endeavors that yield results in other countries (Wilson and Blank 1999; Ma 1999), on "Learning Mathematics for a New Century" (Burke and Curcio 2000), and so on.

Findings from research have been put to use in classrooms (Fennema and Romberg 1999) and have been proven: "The classrooms described . . . provide existence proofs that classrooms that promote understanding can and do exist. . . . Students who studied in them did come to understand mathematics [worthy of understanding]." Continuing, the authors suggest that "a critical question remains: In what ways can many more classrooms be developed so that all students have the opportunity to learn with understanding?" (Fennema, Sowder, and Carpenter 1999, 185). (See Impact of National Science Foundation-funded programs in Appendix A.)

It is clear in an age of continual learning that those who adapt and learn readily and efficiently are advantaged. The growth of technological endeavors, of multinational industries, of e-commerce, and so on are excellent examples of knowledge assimilated quickly and used innovatively. Education, as a rule, does not move quickly to learn or even to capitalize on its own fruits. The effect of this reluctance is that education is often several years behind where the "real world" is. Never has this been more true. A current cartoon suggests, "I don't get it! They make us learn reading, writing, and arithmetic to prepare us for a world of videotapes, computer terminals, and calculators!"

In the United States, a curriculum that has focused on arithmetic, followed by one year of algebra, followed by one year of geometry, often taught using behaviorist methodology has resulted in a fragmented mathematics and has often perpetuated content and methodology without regard to current knowledge. United States mathematics programs are unique in the world in teaching mathematics as separate entities. Other countries teach "maths," using whatever mathematics is needed to solve "realistic" (Fruedenthal 1983) problems that lead students toward building understanding.

A focus on memorized facts and routine procedures, on a curriculum that "design[s] each course primarily to meet the prerequisites of the next course" (Romberg and Kaput 1999, 4); on routine functions that isolate mathematics as a discipline from other "thinking" disciplines; on workbooks, lists of problems, paper-and-pencil manipulations, and so on, has resulted in "a tedious, uninteresting path." There are many hurdles to clear, such as the layer-cake approach described by Steen—a few strands (e.g., arithmetic, algebra, geometry, calculus) arranged horizontally with little opportunity to informally develop intuition along the multiple roots of mathematics (1990, 4). There is unacceptably high attrition from the mathematics program, and there is "little resemblance to what a mathematician or user of mathematics does" (Romberg and Kaput 1999, 5). Nationally there has been concern about our general mathematical performance. Mathematics does not "work" for many citizens. It is not unusual to hear comments such as "Mathematics . . . I hated it . . . never did understand it . . . thank goodness for calculators!" "Innumeracy, an inability to deal comfortably with the fundamental notions of number and chance, plagues far too many otherwise knowledgeable citizens" (Paulos 1988, 3).

Yet when there is talk about changing the mathematics that is taught to address identified needs, conversations often devolve into a skills-versus-problem-solving argument. Wu calls this a "bogus dichotomy in mathematics education. There is not 'conceptual understanding' and 'problem-solving skill' on the one hand and 'basic skills' on the other nor can one acquire the former without the latter." He continues with an illustration that pictures a "violinist who still worries about fingering positions" and "an opera singer without the requisite high notes." (Wu 1999, 14).

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—Wu 1999, 14

Schoenfeld, in *Mathematical Problem Solving*, talks about his revised expectations for problem solving based on in-depth research via videotaped analysis of college mathematics students. Students' resources were far weaker than their performance on standard tests, they had little or no awareness of mathematical heuristics (general problem-solving techniques), they did not know how to deploy the resources (basic mathematical knowledge) they had at their disposal, they were unable to tackle nonstandard problems efficiently, many had serious misunderstandings about mathematics, and they "did not perceive their mathematical knowledge as being useful to them, and consequently did not call upon it" (1985, 13). The disconnect between skills and problem solving was clearly evident.

It is not then a case of teaching skills and procedures or of teaching problem solving. It is a case of teaching both, of making connections, and of building conceptual understanding so that retention and transfer are facilitated. "Learning [with understanding] is generative" (Carpenter and Lehrer 1999, 19). Skills and procedures can be best built when problem-solving situations call for them. Problem solving can best be accomplished when there is automaticity with skills and procedures. Understanding truly surfaces when nonroutine situations are encountered.

Mathematical progress today is dependent upon using what we know about mathematics teaching and learning. It requires programs that are predicated on this knowledge. It calls for teachers who continually strive to know current mathematics techniques congruent with student teaching and learning, and who possess pedagogical content knowledge in sync with such understandings. Assessments need to reflect the content and the manner in which mathematics is taught. Support for such endeavors needs to be enlightened and pervasive.

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Much of the knowledge that exists today did not exist at the time that teachers, parents, and community members attended school. Consequently, "those who are charged with the reform are often those who were most successful with 'the old ways' " (Sparks 1999). If we are to truly make a difference in mathematical understanding, an openness to using what we know about mathematics in this time and place is imperative. This often means changing belief systems … a Herculean task but one of great importance if progress is to be made.

"I have a son who is a doctor. As I visited with him the other evening, he was researching something on his computer. Looking up, he noted, 'You know, Mother, yesterday's knowledge is not sufficient to treat today's patient.'

"Today's children need to benefit from today's knowledge."

—Grunow 2000

I could not help but think, these are children of a new millennium. We are in possession of 'how to' knowledge that has not existed before. Today's children need to benefit from today's knowledge" (Grunow 2000).

Time

> All areas of educational endeavor lament the lack of it. Everyone wishes they had more. Entire studies and books are written about the management of it. We try to cram more and more into less and less of it. We know that time to learn, cogitate, and reflect is an important component of the assimilation of knowledge. We understand that the amount of time needed to learn complex material is roughly proportionate to the complexity of the information. We know that learning time needs to be more than time on task. But we tend to ignore all we know and continue to overschedule to the detriment of real learning.

We tend to ignore all we know [about the time required for complex learning] and continue to overschedule to the detriment of real learning.

Meaningful learning requires time. Students who are building understanding need time to investigate problems in-depth, discuss conjectures with classmates, reflect on their learning, and summarize mathematical understanding. To realistically accomplish these tasks, schools need to examine the amount of time allocated to mathematics classes, ensuring that the time is uninterrupted, and to look at extended timeframe learning opportunities. The "typical" American schedule assigns "an impartial" 51 minutes for each class period (no matter how well or poorly students comprehend the material), spends "only 41 percent of secondary school time on core academic subjects," offers about 5.6 hours of classroom time a day, and runs a 180-day schedule. "Our timebound mentality has fooled us all into believing that schools can educate all of the people all of the time in a school year of 180 six-hour days . . . we ask the impossible of our students. We expect them to learn as much as their counterparts abroad in only half the time" (National Education Commission on Time and Learning, 7).

The standards movement in education has called for more demanding subject matter for all students in all curricular areas. "It is important to be realistic about the amount of time it takes to learn complex subject matter" (Bransford, Brown, and Cocking 1999, 46). Citing a study of algebra students in which regular algebra students received an average of 65 hours of instruction and homework during the year and students in honors algebra received approximately 250 hours of instruction and homework, Singley and Anderson conclude, "in all domains of learning, the development of expertise occurs only with major investments of time, and the amount of time it takes to learn materials is roughly proportional to the amount of material being learned" (Singley and Anderson 1989, 46).

"The amount of time it takes to learn materials is roughly proportional to the amount of materials being learned."

—Singley and Anderson 1989, 46

School learning has other unique ramifications. Klausmeier (1985) notes that learners in school settings are often faced with tasks that do not immediately have meaning or logic. It takes time to explore underlying concepts and to contemplate connections to previous learning before learning with understanding occurs. Pacing is extremely important. There needs to be time for processing information. Pezdek and Miceli (1982) found that third graders could not mentally integrate information delivered in pictorial and verbal formats in 8 seconds; it took 15 seconds to do so. Coverage of topics requires time. If topics are covered too quickly, learning and transfer are hindered. Some students learn only an isolated set of facts and others cannot grasp principles because they do not have enough specific knowledge to make them meaningful. "The implication is that learning cannot be rushed; the complex cognitive activity of information integration requires time" (Bransford, Brown, and Cocking 1999, 46).

Monitoring or using feedback to assess one's progress is an important component of learning.

There is also a difference in learning and transfer in relation to the way instructional time is used. Monitoring or using feedback to assess one's progress is an important component of learning. "Feedback that signals progress in memorizing facts and formulas is different from feedback that signals the state of students' understanding" (Chi et al., 1989, 1994). Additionally, students need to know about the degree to which they know "when, where, and how to use the knowledge they are learning" (Bransford, Brown, and Cocking 1999, 47). Mere chapter and problem completion can give erroneous feedback regarding understanding. Contrasting cases can provide good feedback about learning for both perceptual and conceptual learning (Gibson and Gibson 1955; Bransford et al. 1989). Helping students see potential transfer implications is also useful (Anderson, Reder, and Simon 1996). "While time on task is necessary for learning, it is not sufficient for effective learning. Time spent learning for understanding has different consequences for transfer than time spent simply memorizing facts or procedures from textbooks or lectures. In order for learners to gain insight into their learning and their understanding, frequent feedback is critical: students need to monitor their learning and actively evaluate their strategies and their current levels of understanding" (Bransford, Brown, and Cocking 1999, 66).

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Though volumes have been written about implementing the standards, there is notably little said about the time required to reap the kind of results that are sought. In a report called *Prisoners of Time* (1994), the National Education Commission on Time and Learning calls time "the missing element in the school reform debate" and the "overlooked solution to the standards problem" (9). The commission found five unresolved issues that present insurmountable barriers to efforts to improve learning that have to be addressed before adequate progress can be made.

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- The fixed clock and calendar is a fundamental design flaw that must be changed.
- Academic time has been stolen to make room for a host of nonacademic activities.
- Today's school schedule must be modified to respond to the great changes that have reshaped American life outside the school.
- Educators do not have the time they need to do their job properly.
- Mastering world-class standards will require more time for almost all students.

The commission went on to contend that:

- High-ability students are forced to spend more time than they need on a curriculum developed for students of moderate ability. Many become bored, unmotivated, and frustrated. They become prisoners of time.
- Struggling students are forced to move with the class and receive less time than they need to master the materials. They are pushed on before they are ready, they fall further and further behind, they are penalized with poor grades, and they often ultimately drop out. They, too, are also prisoners of time.
- "Average" students also get caught as teachers try to motivate the gifted and help those in difficulty. Average students are robbed of quality focused time. Typical students are also prisoners of time.

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"The paradox is that the more the school tries to be fair in allocating time, the more unfair the consequences. Providing equal time for students who need more time guarantees unequal results. If we genuinely intend to give every student an equal opportunity to reach high academic standards, we must understand that some students will require unequal amounts of time, i.e., they will need additional time" (National Education Commission on Time and Learning 1994, 15).

"There is a conviction in American schools that the only valid use of teachers' time is 'in front of the class,' that 'reading, planning, collaboration with other teachers and professional development are somehow a waste of time.'"

—NECTL 1994, 17

The commission then contends that the other prisoners of time are the staff. There is a conviction in American schools that the only valid use of teachers' time is "in front of the class," that "reading, planning, collaboration with other teachers and professional development are somehow a waste of time" (17). The National Education Association states, "resolution of the time issue remains one of the most critical problems confronting educators today" (National Education Commission on Time and Learning 1994, 19). A representative of the American Federation of Teachers at a GOALS 2000 U.S. Department of Education Teacher Forum noted that teachers, principals, and administrators need time—to come up to speed as academic standards are overhauled, to come to grips with new assessment systems, to make productive and effective use of greater professional autonomy. If time is not allotted, "it sends a powerful message: don't take reform too seriously. Squeeze it in on your own time" (19).

Some findings regarding teacher changes further illuminate the issue. According to a RAND study (National Education Commission on Teaching and Learning 1994, 17), new teaching strategies can require as much as 50 hours of instruction, practice, and coaching before teachers become comfortable with them. A study of successful urban schools indicates they needed up to 50 days of external technical assistance for coaching and strengthening of staff skills through professional development. Recent professional development studies are finding success when there is a combination of introductory study, learning institutes, and ongoing online follow-up (see exemplary Wisconsin professional development models in Appendix F of this document).

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—Askey 1999, 49

In an article on knowing and teaching elementary mathematics, Richard Askey discusses the grounding that is necessary for teachers to facilitate the development of understanding: "We cannot continue to abandon teachers at every critical stage of their development and then send them into the classroom with a mandate to 'teach for understanding.' This is dishonest and irresponsible. As things stand now, we are asking teachers to do the impossible" (Askey 1999, 49).

Though we must view international studies cautiously because of the many differences involved, there may be some things we can consider as we look at international practices. "For teachers across all countries, time is both a resource and a constraint" (Continuing to Learn from TIMSS Committee 1999, 65). Cursory examination of "instructional time" places the United States in the top half of nine countries examined, but core academic time comparisons show that in the final four years of secondary school, French, German, and Japanese students receive "more than twice as much core academic instruction as American students" (24). Additionally, "among U.S. twelfth-grade advanced mathematics students who were currently taking a mathematics course, a much lower percentage reported receiving five or more hours of mathematics instruction per week than the international average" (National Center for Education Statistics, February 1998, 74). Interestingly enough, other countries do not ignore the co-curricular and extracurricular activities that fill the rest of the school schedule; they offer them at school or at other sites after the academic day.

Research on homework indicates that it does yield results.

—Walberg, Paschal, and Weinstein 1985

Out-of-school learning is also a factor. Japanese students (15 percent in grade four, rising to 50 percent in grade nine) attend *jukus*—private tutorial services that "enrich instruction, provide remedial help, and prepare students for university examinations" (National Education Commission on Time and Learning 1994, 25). *Jukus* exist in conjunction with the Japanese belief that hard work, not ability or aptitude, is the key to meeting high standards. If more time is required for mastery, diligence and application are necessary. German and European students are accustomed to homework, spending two or more hours on it daily. In the United States, only 29 percent report spending as much time on homework. Interestingly, however, more U.S. twelfth-grade advanced mathematics students report being assigned homework three or more times per week than the international average (90 percent as contrasted with 65 percent) (National Center for Education Statistics 1998, 74).

Research on homework indicates that it does yield results. Walberg, Paschal, and Weinstein, in a synthesis of 15 empirical studies on the effects of homework on learning by elementary and secondary students, found the results to be "large and consistent. . . . When homework is merely assigned without feedback from teachers it appears to raise, on average, the typical student at the 50th percentile to the 60th percentile. But, when it is graded or commented upon, homework appears to raise learning from the 50th to the 79th percentile. This graded-homework effect is among the largest ones discovered in educational research literature" (Walberg, Paschal, and Weinstein 1985, 76). In an analysis of instructional practices that contribute to achievement done by the First in the World Consortium, it was stated, "the data also indicate that differences exist in how homework is assigned and used. Few students are more likely than U.S. students to have homework assigned every day and to discuss their completed homework in class" (National Institute on Student Achievement, Curriculum, and Assessment 1999, 31).

Teachers work varies from country to country, but teachers everywhere are very busy and spend much time outside of school… preparing.

An analysis of teacher time is also of interest. Teachers' work differs from country to country, but teachers everywhere are very busy and spend much time outside of school preparing and grading tests, reading and grading student work, planning lessons, meeting with students and parents, engaging in professional development, reading, keeping records, and completing administrative tasks. Studies done in the 1990s are showing that U.S. teachers spent 2.2 hours per week outside of the school day preparing or grading tests, 2.5 hours planning lessons, and 3.5 hours recordkeeping and doing administrative tasks. Japanese teachers spent 2.4 hours on tests, 2.7 hours on lesson plans, and 4 hours on recordkeeping and administrative tasks outside the school day. Japanese teachers have an eight- to nine-hour official workday and a 240-day school year. U.S. teachers have a seven- to eight-hour workday and an approximately 180-day school year. German teachers have a 5- to 5.5-hour school day and an approximately 184-day school year. Japanese teachers have a broad range of in-school responsibilities, "including cleaning of a portion of the school each day" (Stevenson and Nerison-Low 1997, 127–133).

Japanese teachers generally deal with more students in each classroom, but teach fewer classes—usually four hours a day. The rest of the time is spent in reviewing the day's lesson, preparing for the next day, and collaborating with colleagues. Japanese teachers prepare their lessons and teach them to one another, asking for critiques to improve the lessons. German teachers teach only 21 to 24 hours a week. Non-classroom time is spent on preparation, grading, in-service education, and consulting with colleagues, though German teachers teach in the morning and return to their homes shortly after midday. In Japan and Germany, "teachers are held to high standards" (National Education Commission on Time and Learning 1994, 26).

The National Education Commission on Time and Learning offers eight recommendations to help put time at the top of the nation's reform agenda:

- Reinvent schools around learning, not time.
- Fix the design flaw: Use time in new and better ways.
- Establish an academic day.
- Keep schools open longer to meet the needs of children and communities.
- Give teachers the time they need.
- Invest in technology.
- Develop local action plans to transform schools.
- Share the responsibility: Finger pointing and evasion must end. (29)

Elementary students should study mathematics for at least an hour a day; middle-grades and high school students should "be required to study the equivalent of a full year of mathematics in each grade."

—Standards 2000 Project 2000, 371

The National Council of Teachers of Mathematics states in its *Principles and Standards for School Mathematics*, "Learning mathematics with understanding requires consistent access to high-quality mathematics instruction." Elementary students should study mathematics for at least an hour a day "under the guidance of teachers who enjoy mathematics and are prepared to teach it well." Middle-grades and high school students should "be required to study the equivalent of a full year of mathematics in each grade" (four years of high school mathematics). "All middle-grades and high school students should be expected to spend a substantial amount of time every day working on mathematics outside of class, in activities ranging from typical homework assignments and projects to problem solving in the workplace" (Standards 2000 Project 2000, 371).

The challenge is, of course, for the mathematics to be worthwhile and challenging, for the delivery to facilitate development of understanding and be motivating, and for assessment of mathematical endeavors to be aligned with content and delivery and gleaned from multiple, authentic measures.

Equity

> Mathematical equity is provision of a mathematics education for *all* students that: empowers them to function mathematically in our society; helps them become confident in their abilities to "do" mathematics; leads them to become problem solvers of problems of all natures; and fits them for further mathematical endeavors. In an era of achievement, "because the great masses of students need to be educated for thinking for work rather than for low-skilled tasks, and educational success is a necessity rather than a luxury for a chosen few . . . schools are now expected to ensure that all students learn and perform at high levels" (Darling-Hammond, Wise, and Klein 1999, 2). Yet, mathematics as a discipline has been one of the least equitable. There is more tracking in mathematics than any subject area. Mathematics, more than any discipline, has been used to sort and separate students, often removing life choices in accordance with the results. No discipline needs examine equity more than mathematics!

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In 1989, the NCTM *Curriculum and Evaluation Standards* stated that "equity has become an economic necessity." Making an impassioned plea for mathematical literacy for all, the standards characterized mathematics as a "critical filter for employment and full participation in our society. We cannot afford to have the majority of our population mathematically illiterate" (Commission on Standards for School Mathematics 1989, 4). The NCTM *Professional Standards for Teaching Mathematics* reemphasized that the "most compelling goal" is "the comprehensive mathematics education of every child . . . *all students*." "Every child" was then identified more specifically—"students who have been denied access in any way to educational opportunities as well as those who have not; students who are African American, Hispanic, American Indian, and other minorities as well as those who are considered to be a part of the majority; students who are female as well as those who are male; and students who have not been successful in school and in mathematics as well as those who have been successful." It was concluded that "it is essential that schools and communities accept the goal of mathematical education for *every* child" (Commission on Teaching Standards for School Mathematics 1991, 4). Continuing with the theme, the NCTM *Assessment Standards for School Mathematics* specified an equity standard. "In the past we wanted all students to learn some mathematics, but we differentiated

"We cannot afford to have the majority of our population mathematically illiterate."

—Commission on Standards for School Mathematics 1989, 4 among the types of mathematics education different groups of students received. Now we have high expectations for all students, envisioning a mathematics education that develops each student's mathematical power to the fullest." The standards then elucidated, "In an equitable assessment, each student has an opportunity to demonstrate his or her mathematical power. Because different students show what they know and can do in different ways, assessments should allow for multiple approaches" (Assessment Standards Working Groups 1995, 15).

The NCTM *Principles and Standards for School Mathematics* offer an equity principle: "Excellence in mathematics education requires equity—high expectations and strong support for all students" (Standards 2000 Project 2000, 11). Contending that "the vision of equity in mathematics education challenges a pervasive societal belief in North America that only some students are capable of learning mathematics," (12) the principles continued:

- Equity requires high expectations and worthwhile opportunities for all.
- Equity requires accommodating differences to help everyone learn mathematics.
- Equity requires resources and support for all classrooms and all students (12–13).

"Equity does not mean that every student should receive identical instruction."

—Standards 2000 Project 2000, 12

It was carefully explained that "equity does not mean that every student should receive identical instruction; instead, it demands that reasonable and appropriate accommodations be made as needed to promote access and attainment for all students" (12).

L. A. Steen, in *Why Numbers Count: Quantitative Literacy for Tomorrow's America*, compellingly points out, "The relentless quantification of society continues unabated. The tendency to reduce complex information to a few numbers is overwhelming—in health care, in social policy, in political analysis, in education. . . . Literacy is about . . . solving problems and using technology; it is about practices as well as knowledge, procedures as well as concepts. Numbers count because ideas count" (1997, xxvii). The thrust of the plea is that to survive successfully in today's world, all citizens need to possess "numeracy."

To acquire mathematical proficiency, all students obviously need access to opportunities that will help them build understanding.

To acquire mathematical proficiency, all students obviously need access to opportunities that will help them build understanding. In the United States it is common practice to "label" and "sort" students. "The practice of dividing students into instructional groups on the criterion of assumed similarity in ability or attainment is widespread . . . from assigning them temporarily to separate groups within a single classroom to setting up classes in differentiated streams or tracks. Students may be streamed only for various subjects or for their entire range of school-based learning" (Oakes 1985, ix). Whether this practice is done to ensure continued challenge for the best and brightest, to facilitate a reduction in student variability for the management and teaching of a group, or to bring together students of like dispositions, research continues to show that the practice is extremely inequitable. Oakes looked at the results from John Goodland's University of California at Los Angeles study of thirty-eight schools, "A Study of Schooling," and found that "there were clear differences between upper and lower tracks in regard to the content and quality of instruction, teacher-student and student-student relationships, the expectations of teachers for their students, the affective climate of classrooms, and other elements of the educational enterprise. It appears that those students for whom the most nurturant learning would appear to be appropriate received the least. Not only do individual schools differ widely in the quality of education they provide, but also, it appears, quality varies substantially from track level to track level within individual schools" (1985, xi).

There [are] clear differences between upper and lower tracks in regard to the content and quality of instruction.

—Oakes 1985, xi

Additionally, minority students were overrepresented in the lower tracks, as were white children of low-income families. Vocational programs served as forms of tracking in which poor and minority children were overrepresented. Surprisingly, gifted and talented students often were "not well provided for in upper tracks, just as slower students are not well provided for in the lower tracks" (Oakes 1985, xii). Indeed, gifted mathematics students frequently handle memorization and procedures so well that it is assumed that they know the mathematics concepts represented. They are not given time to "muck about" with the mathematics (Phillips 1987) and to build understandings. They suffer perhaps more than students who are given time and with whom innovative approaches are used.

"Evidence suggests that teachers themselves are tracked, with those judged to be the most competent and experienced assigned to the top tracks" (Darling-Hammond 1997, 269). In a study done in Chicago, "Closing the Divide," Dreeben (1987) found that outcomes among students were explained not by socioeconomic status, race, or ability levels but by the quality of instruction . . . having enough time, covering a substantial amount of rich curricular materials, matching instruction appropriately to the ability levels of groups. Across the board, experimental studies offer strong evidence that "what students learn is substantially a function of the opportunities they are provided." Likewise, it has also become clear that "differences in teacher expertise are a major reason for the differences in learning opportunities across schools and classrooms" (Darling-Hammond 1997, 271).

Resources matter.

Additionally, resources matter. In a large-scale study of more than one thousand school districts, Ferguson (1991) found that money matters in education. Student achievement increases with expenditure levels. The single most important determinant of student achievement is teacher expertise and experience. Small student–teacher ratios are statistically significant determinants of students outcomes. "Knowledgeable teachers working in personalized settings are the most important key to learning" (Ferguson 1991, 490). Where teachers are poorly trained and class sizes are large, students suffer from worksheets, rote drill, superficial texts, sitting for long periods of the day, working on boring tasks of low cognitive level, and seldom talking about what they know or constructing and solving problems that involve higher thinking skills (Metz 1978). Equalization of resources for school funding is needed to ensure equity.

Fennema and Leder define* equity *as equal opportunity to learn mathematics, equal educational treatment, and equal educational outcomes.

Fennema and Leder, in *Mathematics and Gender,* define *equity* as equal opportunity to learn mathematics, equal educational treatment, and equal educational outcomes (1990, 3–4). They contend that although justice for both females and males has been an underlying belief in mathematics education, studies have been based on the use of a male standard, females' achievement and beliefs about themselves in comparison with those of males. Female contributions to mathematics are minimized because of attitudes toward females as mathematicians. Classrooms perpetuate these notions. When students construct knowledge, the mental activity that takes place is as important as overt behavior. Knowledge that is brought to the activity, confidence in approaching the task at hand, control felt in doing the activity, interest in the task—all affect the quality of participation. These autonomous learning behaviors are important in working with high-cognitive-level activities. Fennema and Leder suggest that females have less opportunity to develop autonomous learning behaviors (ALBs): "Teachers often interact differently with their males and female students, with males attracting more and qualitatively different interactions" (1990, 17). Females express lower confidence in their ability to do mathematics, often suffer negative consequences such as unpopularity when they achieve mathematical success (Clarkson and Leder 1984), and are uncertain about the appropriateness of doing mathematics (Boswell 1985); females feel they have to work harder to attain success in mathematical endeavors (Fennema 1985), and these "attributions of success" often lead to cognitive, motivational, and/or emotional deficits (Weiner 1980). The "practices, values, expectations, and beliefs of both individuals and society must be examined if the currently existing gender differences in mathematics participation and performance are to be understood and changed" (Fennema and Leder 1990, 21).

Autonomous learning behaviors are important in working with high-cognitive-level activities.

Secada and Berman (1999) suggest interestingly that the present emphasis on teaching for understanding raises some questions. Context is important for the connections that develop in building understanding. It is important that those contexts do connect with student backgrounds and match the diversity of the students who are learners of mathematics. Extension of mathematics into different domains also needs to reflect domains with which diverse students are familiar. In classrooms that promote understanding, students are encouraged to talk about thinking used in solving problems. This opens student thinking to analysis; differences become clearer. Equitable classrooms work with those differences to position them as strengths rather than as unenlightened positions. Sharing of solution strategies also opens students to scrutiny. In classrooms where the substance of the solution is valued, widespread participation is valued, appreciation for diversity in thought is expressed, and the merit of the strategy rests on the mathematics, not the doer of the mathematics, students feel free to offer their findings and to open them to discussion.

Equitable classrooms work with differences to position them as strengths rather than as unenlightened positions.

Students are likewise encouraged to question, to assume responsibility for their own learning, and to share in the "mathematical authority" (Secada and Berman 1999, 37) afforded by such discourse. If this classroom norm differs considerably from home behavior expectations, it can create an uncomfortable dichotomy for students. Classrooms that honor equity work with autonomous components to make them work for the good of understanding and the students. Hailing the emphasis on building understanding for each student as a noble goal, Secada and Berman suggest that care in implementation must be taken to make sure that all students do have an opportunity to construct knowledge and that those who facilitate such knowledge building be cognizant of the possible inequities in application that could arise.

NCTM carefully points out that rich mathematics for all does not mean "sameness" for all. Cognitively Guided Instruction, a philosophy of mathematical knowledge building, is built on listening to student cognitions to inform teacher decision making and curriculum implementation (Fennema, Carpenter, and Peterson 1989). Some of the outstanding premises of the *Professional Standards for Teaching Mathematics* (Commission on Teaching Standards for School Mathematics 1991) are the affirmations of teachers as curriculum decision makers, as selectors of worthwhile mathematical tasks, as facilitators of discourse for building understanding and for listening to students' cognitions, and as creators of environments that foster construction of knowledge. Thus, teachers listen to students and develop the curriculum in conjunction with classroom and student readiness, picking up where students are and moving forward appropriately. What could be more equitable than continual progress based on expressed understanding?

What could be more equitable than continual progress based on expressed understanding?

Building on student understanding also implies differentiation. "Teachers must be ready to engage students in instruction through different learning modalities, by appealing to differing interests, and by using varied rates of instruction along with varied degrees of complexity" (Tomlinson 1999, 2). Acknowledgement of student learning diversities has prompted *Teaching with the Brain in Mind* (Jensen 1998); *Frames of Mind: The Theory of Multiple Intelligences* (Gardner 1983); *A Different Kind of Classroom: Teaching with Dimensions of Learning* (Marzano 1992); *Cognitive Type Theory and Learning Style* (Mamchur 1996); *Concept-Based Curriculum and Instruction: Teaching beyond the Facts* (Erickson 1998); and *Discovering and Exploring Habits of Mind* (Costa and Kallick 2000). If knowledge is to be fed into complex weblike structures and multiple connections, then it would seem logical to address knowledge building through many "representations" and translations between and among them (Post et al. 1993). Selection and use of representations was thought so important that NCTM added it as a fifth process standard PSSM: "By listening carefully to students' ideas and helping them select and organize representations that will show their thinking, teachers can help students develop the inclination and skills to model problems effectively, to clarify their own understanding of a problem, and to use representations to communicate effectively with one another" (Standards 2000 Project 2000, 209).

"Teachers must be ready to engage students in instruction through different learning modalities, by appealing to differing interests, and by using varied rates of instruction along with varied degrees of complexity."

—Tomlinson 1999, 2

Equity is a goal of all mathematics programs. Consequently, disparity is cause for concern. In the Council of Chief State School Officers publication *State Indicators of Science and Mathematics Education 1999*, this analysis is given: "All states have a significant disparity between the percent of white students at or above the Basic level and the percentage for the largest minority group" (Blank and Langesen 1999, 16). In mathematics disparity, the Wisconsin state difference at the eighth grade level between white and black students in percent at or above Mathematics Basic level is 63 percentage points; between white and Hispanic students, 37 points. The disparity between white and minority students increased from 1992 to 1996 by 19 points. Still, it needs to be noted that the trend for all three populations from 1982 to 1994 has been upward; all populations are making proress.

Attention to closing gaps and to disaggregating data to determine strengths and weaknesses does seem to be indicating progress on many fronts. On the new main National Assessment of Educational Progress (NAEP), gender differences on math scores have almost disappeared in elementary and middle school and, though high school girls continued to lag behind boys on the Third International Mathematics and Science Study (TIMSS), there is less difference in Grade 12 scores between males and females than there is on the long-term trend (*Education Week on the Web,* 21 June 2000; www.edweek.org/ew/ewstory.cfm?slug=41tl.h19). Math scores are rising across the country at a national average rate of about one percentile point per year, "a pace outstripping that of the previous two decades and suggesting that public education reforms are taking hold . . . [though] "progress is far from uniform" (Rand News Release, 25 July 2000; www.rand.org/publications/mr/mr92k). Finding Wisconsin to be second in achievement progress by students from similar families, the factors that seemed to contribute to the differences were lower pupil–teacher ratios, higher participation in public prekindergarten programs, and a higher percentage of teachers who are satisfied with the resources they are provided for teaching. Low teacher turnover also seemed to contribute.

Math scores are rising across the country at a national average rate of about one percentile point per year, "a pace outstripping that of the previous two decades and suggesting that public education reforms are taking hold."

—Rand New Release, July 2000, 25

Another analysis of NAEP data found that rural students show "highly comparable levels of achievement relative to their nonrural counterparts" (Lee and McIntire 2000, 3) and, in fact, "outperform nonrural students" on the most recent testing in 1996. Given that many rural students are poor and attend schools whose instructional resources and course offerings are limited, the level of their academic performance relative to their nonrural counterparts is encouraging. Indeed, the rural schools can provide a model of strength worth studying and emulating. The factors that seem to give the competitive edge are the better social/organizational context that includes teacher training, safe and orderly climate, and collective support.

Another contributing factor is that more students are taking "tough" math (Viadero 2000, 8; www.edweek.org; Cole 2000, 5; www.ccsso.org). In math, 63 percent of students took three years of high school mathematics, compared with 49 percent in 1990. The proportion of students taking tough math courses such as advanced placement calculus, third-level algebra, and analytical geometry increased from 25.2 percent in 1982 to 41.4 percent in 1998. Nationally, 5 percent of students took advanced placement math exams. "We're coming back to what is important for students to know," according to Christopher T. Cross, president of the Council for Basic Education (Viadero 2000, 8).

More students are taking 'tough' math.

—Viadero 2000, 8

Noting that "equity is a critical factor in the nation's economic viability," Croom, in discussing mathematics for all students, suggests that "eliminating the social injustices of past schooling practices will require the support of policymakers, administrators, teachers, parents, and others concerned about excellence and equity in mathematics education." He continues: "All children can learn challenging mathematics with appropriate support and an equitable learning environment" (Croom 1997, 7). This concern and belief have been evidenced in current legislation. The Improving America's Schools Act (1994), a reauthorization of the Elementary and Secondary Education Act (ESEA) of 1965, called for challenging content for all students, for identification of students who are not proficient, and for efforts to move those students through adequate yearly progress measures to proficiency. Current state assessment measures, the *Wisconsin Student Assessment System*, has responded to this national call for such identification, and coupled with state legislation, 115.38, that asked for identification of schools in need of improvement and assistance to those schools to facilitate growth, Wisconsin continues to try to assist its school districts in providing an education of excellence for *all* Wisconsin students.

Technology

> In a discussion of technology-enriched learning of mathematics, Wattenberg and Zia state, "It is easy to feel like kids in a candy store as each new year brings new and more exciting technology for doing and learning science and mathematics—personal computers, scientific and graphing calculators, calculators and computers with computer algebra systems linked to devices for collecting scientific data, the World Wide Web, Java applets, and so much more" (Wattenberg and Zia 2000, 67). There is increasing access to primary resources, massive and real-time data sets, and museum-quality collections. "Mathematics with its tools for analysis and visualization is at the very center of the best technology-enriched, inquiry-driven learning, and mathematical modeling is especially important to enable our students to understand and build the sophisticated and compelling simulations that are becoming so common and important. Yet, questions remain: What tools have been shown to improve our students' learning? When and how do we use them?" (68).

"Mathematics with its tools for analysis and visualization is at the very center of the best technology-enriched, inquiry-driven learning."

—Wattenberg and Zia 2000, 68

In the foreword to *Standards for Technological Literacy: Content for the Study of Technology* (Technology for All Americans Project 2000), the contention is made that "we are a nation increasingly dependent on technology. Yet, in spite of this dependence, U.S. society is largely ignorant of the history and fundamental nature of the technology that sustains it" (v). Technology is defined (broadly speaking) as "how people modify the natural world to suit their own purposes." An analysis of the technology content standards indicates that study of the various components could certainly enhance understanding of the role of educational technology in mathematical learning and could enlighten debate regarding the use of technological tools in mathematics.

Technology is defined (broadly speaking) as "how people modify the natural world to suit their own puposes."

—Technology for All Americans Project 2000, v

The technology content standards are:

- Students will develop an understanding of The Nature of Technology. This includes acquiring knowledge of: (1) the characteristics and scope of technology; (2) the core concepts of technology; and (3) the relationships among technologies and the connections between technology and other fields.
- Students will develop an understanding of Technology and Society. This includes learning about: (4) the cultural, social, economic, and political effects of technology; (5) the effects of technology on the environment; (6) the role of society in the development and use of technology; and (7) the influence of technology on history.

- Students will develop an understanding of Design. This includes knowing about: (8) the attributes of design; (9) engineering design; and (10) the role of troubleshooting, research and development, invention and innovation, and experimentation in problem solving.
- Students will develop Abilities for a Technological World. This includes being able to: (11) apply the design process; (12) use and maintain technological products and systems; and (13) assess the impact of products and systems.
- Students will develop an understanding of The Designed World. This includes selecting and using: (14) medical technologies; (15) agricultural and related biotechnologies; (16) energy and power technologies; (17) information and communication technologies; (18) transportation technologies; (19) manufacturing technologies; and (20) construction technologies.

Technology allows students to visualize and experience mathematics in heretofore impossible ways.

Mathematics and technology are closely intertwined. Many of the technological inventions of today rest heavily on mathematical inception, mathematics for development, and mathematics for implementation. But the interaction is reciprocal. Much of the mathematical change that has taken place has occurred in response to technological advances. "Technology is an important resource for teaching and learning mathematics. Calculators, computers, and the World Wide Web are invaluable for students and teachers in the classroom. Technology allows students to visualize and experience mathematics in heretofore impossible ways, engage in real-world (rather than contrived) problem solving, perform rapid and complex computations, and generate their own representations of their own learning. Furthermore, technology allows students to undertake projects that connect with global communities, integrate mathematics with other subjects, and fit students' individual needs and interests" (International Society for Technology in Education, 2000, p. 96).

Technology enhances mathematics learning; technology supports effective mathematics teaching; and technology influences what mathematics is taught.

—Standards 2000 Project 2000, 24–27

Principles and Standards for School Mathematics calls electronic technologies—calculators and computers—essential tools for teaching, learning, and doing mathematics. Research on the appropriate use of technology shows that students can expand mathematical knowledge and conceptual understandings with the use of technology (Boers-van Oosterum 1990; Sheets 1993; Groves 1994; Dunham and Dick 1994). Noting that "technology should not be used as a replacement for basic understanding and intuitions," but to "foster those understandings and intuitions," the *standards* urge "responsible" use of technology to further mathematical understanding. Three points are then made: (1) technology enhances mathematics learning, (2) technology supports effective mathematics teaching, and (3) technology influences what mathematics is taught (Standards 2000 Project 2000, 24–27).

"Students using calculators tended to have better attitudes toward mathematics… better self-concepts… and there was no loss in student ability to perform paper-and-pencil computational skills."

—Hembree and Dessart 1992, 129

Hembree and Dessart, in a meta-analysis of 79 non-graphing calculator studies, found "improvement in students' understanding of arithmetical concepts and in their problem-solving skills . . . students using calculators tended to have better attitudes toward mathematics . . . better self-concepts . . . and there was no loss in student ability to perform paper-and-pencil computational skills when calculators were used as part of mathematics instruction" (Hembree and Dessart 1992, 129). Research on the use of scientific calculators with graphing capabilities "also has shown positive effects on student achievement." The positive effects were many: improvements in graphing ability, conceptual understanding of graphs, the ability to relate graphical representations to other representations, function concepts, spatial visualization, problem solving, flexibility in thinking, perseverance, and focus on conceptual understanding. Students who work with graphing calculators are more likely to solve problems graphically than use other methods such as algebra. "Studies of graphing calculators have found no negative effect on basic skills, factual knowledge, or computational skills" (Hembree and Dessart 1992, 129).

Research on the use of scientific calculators with graphing capabilities "also has shown positive effects on students achievement."

—Hembree and Dessart 1992, 129

In a report of a study done by the Policy Information Center, Research Division, for the Educational Testing Service, some caveats regarding the positive results of technological use are offered. The national study of the relationship between different uses of educational technology and various educational outcomes drew data from the 1996 National Assessment of Educational Progress (NAEP) in mathematics and used samples from 6,227 fourth-graders and 7,146 eighth-graders. The study examined information on frequency of computer use for mathematics in the school, access to computers and frequency of computer use in the home, professional development of mathematics teachers in computer use, and kinds of instructional uses of computers by mathematics teachers and their students. The results found that technology is a factor in academic achievement, but that results depend on how it is used. "When computers are used to perform certain tasks, namely applying higher order concepts, and when teachers are proficient enough in computer use to direct students toward productive uses more generally, computers do seem to be associated with significant gains in mathematics achievement, as well as an improved social environment in the school" (Policy Information Center, Research Division, Educational Testing Service 1998, 32).

Technology is a factor in academic achievement, but the results depend on how it is used.

The technology standards contend that technology is not always understood by an accessing public. Waits and Demana, who have pioneered the use of technology in the classroom, share some of their reflections regarding the use of calculators in mathematics. They learned: (1) change can occur if we put the potential for change in the hands of everyone (the handheld calculator); (2) it takes practiced teachers to change the practice of teachers (teachers need to learn how to use technology); (3) calculators cause changes in the mathematics we teach; and (4) calculators cause changes in the way we teach and in the way students learn (Waits and Demana 2000). Elaborating, Henry Pollak, regarding the effect of technology, is quoted:

- Some mathematics becomes less important (paper-and-pencil calculation and symbol-manipulation techniques).
- Some mathematics becomes more important (discrete mathematics, data analysis, parametric representations, and nonlinear mathematics).
- Some mathematics becomes possible (fractal geometry, predictive statistics) (Pollak 1986).

Additionally, Waits and Demana found:

- Calculators reduce the drudgery of applying arithmetic and algebraic procedures when those procedures are not the focus of the lesson.
- Calculators with computer interactive geometry allow for investigations that lead to a much better understanding of geometry (Vonder Embse and Engebretsen 1996).
- Calculators help students see that mathematics has value, interest, and excitement (Bruneningsen and Krawiec 1998).
- Calculators make possible a "linked multiple-representation" approach to instruction.
- "Before calculators we studied calculus to learn how to obtain accurate graphs. Today we use accurate graphs to help us study the concepts of calculus" (Waits and Demana 2000, 57).

The best use of technology balances the use of paper-and-pencil techniques and technology.

—Waits and Demana 2000, 59

Waits and Demana believe that the best use of technology balances the use of paper-and-pencil techniques and technology. There are "appropriate uses" for both; the two can complement one another. A good balance can be effected by having students:

1. Solve problems using paper and pencil and then support the results using technology.
2. Solve problems using technology and then confirm the results using paper-and-pencil techniques.
3. Solve problems for which they choose whether it is most appropriate to use paper-and-pencil techniques, technology, or a combination of both.

"These approaches help students understand the proper use of technology" (Waits and Demana 2000, 59). (Appropriate use of manipulatives was also urged by Waits and Demana.)

The National Council of Teachers of Mathematics has offered two position statements titled "Calculators and the Education of Youth" and "The Use of Technology in the Learning and Teaching of Mathematics" (see Appendix G).

"What has not yet been fully understood is that computer-based technologies can be powerful pedagogical tools . . . extensions of human capabilities and contexts for social interactions supporting learning.

—Bransford, Brown, and Cocking 1999, 218

Bransford, Brown, and Cocking believe that technology has a broad role to play in education. "What has not yet been fully understood is that computer-based technologies can be powerful pedagogical tools . . . extensions of human capabilities and contexts for social interactions supporting learning" (Bransford, Brown, and Cocking 1999, 218). As such, technology resources for education function in a social environment, "mediated by learning conversations with peers and teachers." Kozma and Schank (1998) concur, "emphasis in U.S. schools [has been] on individual learning and performance—what students can do by themselves" (Kozma and Schank 1998, 3). The twenty-first century, they continue, will make much different demands. "Symbolic analysts (Reich 1991)—problem identifiers, problem solvers, strategic brokers" will be needed. They will need a much different set of skills. "They will need to be able to use a variety of tools to search and sort vast amounts of information, generate new data, analyze them, interpret their meaning, and transform them into something new." Communities of learning will be needed to support development of such skills. "In our vision of communities of understanding, digital technologies are used to interweave schools, homes, workplaces, libraries, museums, and social services to integrate education into the fabric of the community" (Kozma and Schank 1998, 5).

"In our vision of communities of understanding, digital technologies are used to interweave schools, homes, workplaces, libraries, museums, and social services to integrate education into the fabric of the community."

—Kozma and Schank 1998, 5

Service Learning

> Service Learning is a method by which young people learn and develop through active participation in thoughtfully organized service experiences. These services meet actual community needs; are coordinated in collaboration with the school and community; are integratd into each young person's academic curriculum; provide structured time for young people to think, talk, and write about what they did and said during the service project; provide young people with opportunities to use newly acquired academic skills and knowledge in real life situations in their own communities; enhance what is taught in the school by extending student learning beyond the classroom; and help foster the development of a sense of caring for others (Alliance for Service-Learning in Education Reform (ASLER) 1998). Mathematics is best learned in meaningful contexts. Certainly the situations proposed in service learning have potential for meaningful mathematics explorations.

The situations proposed in service learning have potential for meaningful mathematics explorations.

The Wisconsin Department of Public Instruction Strategic Plan (2000) identifies among its goals:

GOAL 3. HELP ALL STUDENTS BECOME CARING, CONTRIBUTING, RESPONSIBLE CITIZENS.

3.5 Classroom instruction is enhanced by offering students a variety of opportunities for meaningful participation such as student clubs, service learning, music, athletics and drama.

3.6 Schools will infuse citizenship knowledge and skills across the curriculum.

The U.S. Department of Labor (Wisconsin Department of Public Instruction 1998a, 4) lists the following characteristics that employers look for in teens: listening-to-learn skills; listening and communication; **adaptability**: creative thinking and **problem solving**, especially in response to barriers/obstacles; personal management: self-esteem, goal-setting/self-motivation, personal career development/goals, pride in work accomplished; group effectiveness: interpersonal skills, negotiation, teamwork; organizational effectiveness and leadership; making a contribution; and competence in reading, writing, and **computation**.

The Wisconsin Department of Public Instruction citizenship guide, *Citizenship Building a World of Good, A Tool Kit for Schools and Communities* lists seven characteristics of successful schools in developing caring, contributing, productive, and responsible citizens:

- Core values
- Safe and orderly places
- **Family and community involvement**
- Addressing of societal values
- Positive relationships
- **Engagement of students' minds**
- **High expectations**

Service learning is "a way of teaching and learning that engages students in active service tied to curriculum."
—Kielsmeier 2000, 652

In an attempt to capitalize on one of the most valuable resources of schools—its own students—service learning has become an important thrust. Kielsmeier defines service learning as "a way of teaching and learning that engages students in active service tied to curriculum" (Kielsmeier 2000, 652). Service learning, he continues, "can transform the idealism of youth into a powerful force for educational change and democratic renewal." Fitting well with Wisconsin's identified goal of caring, contributing, and responsible citizens, service learning is a consideration for all disciplines. How, then, can mathematics capitalize on service learning? How can service learning use mathematics as a resource?

The Essential Elements of Service-Learning (National Service-Learning Cooperative 1998) parallel many of the components necessary for the construction of mathematical knowledge.

The National Service-Learning Cooperative (1998) lists Essential Elements of Service-Learning. It is interesting to note that these elements carefully parallel many of the components necessary for the construction of mathematical knowledge.

In effective service-learning, students are involved in construction of their own knowledge. Construction of mathematical understanding is the thrust of the mathematics program.

1. In effective service learning, there are clear educational goals that require the application of concepts, content, and skills from the academic disciplines and involve students in the construction of their own knowledge.

   Principles and Standards for School Mathematics (Standards 2000 Project 2000, 20) states, "research on the learning of complex subjects such as mathematics has solidly established the important role of conceptual understanding in the knowledge and activity of persons who are proficient … along with factual knowledge and procedural facility (Bransford, Brown, and Cocking 1999).

   Effective mathematics learning is facilitated by identification of concepts that need to be developed at specific stages and by alignment of curricula with the envisioned goals. Conceptual development is a focus of effective mathematics programs (Erickson 1998). Students need to be able to identify the mathematics content that is being addressed in explorations in which they engage. Skills need to be developed in conjunction with meaningful problem solving. Construction of mathematical understanding is the thrust of the mathematics program.
2. In effective service learning, students are engaged in tasks that challenge and stretch them cognitively and developmentally.

Mathematics is best learned when the context in which a problem is situated is meaningful and pertinent, when the task at hand is engaging and motivating, when the cognitive level is demanding, when the task calls for the use of previous knowledge in building new understandings, and when the problem moves the student to new, deeper, or broader understandings.

In effective service learning, students are engaged in tasks that challenge and stretch them cognitively and developmentally. Mathematics is best learned when the cognitive level is demanding.

At a National Service-Learning Leader School, Malcolm Shabbaz, an alternative high school located in Madison, Wisconsin, more than half of the 2,600 students participate in activities that combine academic learning and service (Riley and Wofford 2000, 670). Working for local service projects such as raising funds for a respite center for children caught in situations of family violence, students use their talents and skills to benefit others. Students also travel to communities in Appalachia, the Mississippi Delta, and Native American reservations to gain in-depth knowledge of the history and culture of the peoples living there, along with opportunities to repair the homes of senior citizens and work with Head Start classes in those locales.

Noting that "many children from all backgrounds do not understand mathematics enough to use it or cannot even do many tasks accurately," *Toward a Mathematics Equity Pedagogy* (TMEP), an approach based on research on children's thinking and the best from traditional pedagogy and powerful elements of reformed teaching, was developed, has been used, and is reaping successes (Fuson, LaCruz, Smith, LoCicero, Hudson, Ron, and Steeby 2000). There are six aspects of TMEP:

1. Start where children are and keep learning meaningful. Use many meaning-focused classroom activities.
2. Set high-level mathematical goals and expectations for all. Bring children up to the higher mathematics.
3. Develop a collaborative math talk culture of understanding, explaining, and helping.
4. Build on the best of traditional instruction.
5. Facilitate the learning of general school competencies. Increase self-regulatory actions so that children become more organized, understand what they do not understand, are involved in setting some learning goals, and are helped to reflect on their own progress.
6. Mobilize learning help in the home.

3. In effective service learning, assessment is used as a way to enhance student learning as well as to document and evaluate how well students have met content and skill standards.

In effective service learning, assessment is used as a way to enhance student learning. In good mathematics programs, assessment is used to determine where students are in understanding of the topic at hand in order to make curricular decisions regarding the next mathematical tasks to be introduced.

The diverse purposes for which mathematics assessments are used are characterized in the *Assessment Standards for School Mathematics* (Commission on Teaching Standards for School Mathematics 1995, 25) as monitoring students' progress to promote growth, making instructional decisions to improve instruction, evaluating students' achievement to recognize accomplishments, and evaluating programs to modify them.

In good mathematics programs, assessment is used to determine where students are in understanding of the topic at hand in order to make curricular

In effective service-learning, students are engaged in service tasks that have clear goals and significant consequences for themselves and others. Worthwhile Mathematical Tasks, tenets of worthy mathematics programs, represent mathematics as an ongoing human activity.

decisions regarding the next mathematical tasks to be introduced. Good mathematics assessments identify what students know (as opposed to what they don't know!) individually and as a class. "Assessment occurs at the intersection of the important mathematics that is taught with how it is taught, what is learned and how it is learned" (Assessment Standards Presentation Package 1995).

4. In effective service learning, students are engaged in service tasks that have clear goals, meet genuine needs in the school or community, and have significant consequences for themselves and others.

The *Professional Standards for Teaching Mathematics* (1991, 25) offer as Standard 1, Worthwhile Mathematical Tasks. Those tasks should be based on:

- sound and significant mathematics;
- knowledge of students' understandings, interests, and experiences;
- knowledge of the range of ways that diverse students learn mathematics;

and should

- engage students' intellect;
- develop students' mathematical understandings and skills;
- stimulate students to make connections and develop a coherent framework for mathematical ideas;
- call for problem formulation, problem solving, and mathematical reasoning;
- promote communication about mathematics;
- represent mathematics as an ongoing human activity;
- display sensitivity to, and draw on, students' diverse background experiences and dispositions;
- promote the development of all students' dispositions to do mathematics.

Service-learning tasks, because of the rich contexts in which they are situated, challenge students to employ unique thinking, first of all, in recognizing important problems, and then in formulating solution plans and in analyzing how solution steps should be taken. The mathematics that is demanded in service-learning tasks often involves higher-level thinking skills such as statistical analyses and projections. When solution strategies have been employed, reflection on results is important. Often, service-learning tasks require another component—the need to convince others of a possible best solution. Such tasks are indeed worthwhile mathematical tasks.

In effective service learning, formative and summative evaluation is employed. Rich mathematical tasks ask students to formulate a plan and to check solutions for reasonableness of results.

5. In effective service learning, formative and summative evaluation are employed in a systematic evaluation of the service effort and its outcome.

The *Assessment Standards for School Mathematics* (1995, 4) suggests that the assessment process can be thought of as four interrelated phases that highlight principal points at which critical decisions need to be made: plan assessment, gather evidence, interpret evidence, and use results. The *standards* suggest that these phases are not necessarily sequential and that each phase has decisions and actions that occur within the phase.

Rich mathematical tasks ask students to formulate a plan for solving the problem. Execution then often involves reconsidering the approach to the problem, a sifting and winnowing so as to speak of solution strategies. When the problem is solved, checking of the solution for reasonableness against expected results is necessary. The impact of the solution is evaluated. If there is a discrepancy in the result and the expected outcome, reexamination of approach and of the solution is in order. Likewise, the result often requires explanation and defense of result.

6. In effective service learning, student voice is maximized in selecting, designing, implementing, and evaluating the service project.

Standard 3 of the *Professional Teaching Standards* (1991, 45) is the Student's Role in Discourse. It proposes that classroom discourse should be promoted so that students:

- listen to, respond to, and question the teacher and one another;
- use a variety of tools to reason, make connections, solve problems, and communicate;
- initiate problems and questions;
- make conjectures and present solutions;
- explore examples and counterexamples to investigate a conjecture;
- try to convince themselves and one another of the validity of particular representations, solutions, conjectures, and answers; and
- rely on mathematical evidence and argument to determine validity.

In effective service learning, student voice is maximized. In the mathematics classroom, student discourse is used to socially construct mathematical understanding.

The *Wisconsin Model Academic Standards,* (1998, 5) in "Standard A. Mathematical Processes" ask students to "develop effective oral and written presentations employing correct mathematical terminology, notation, symbols, and conventions for mathematical arguments and display of data . . . [and] to organize work and present mathematical procedures and results clearly, systematically, succinctly, and correctly."

7. In effective service-learning, diversity is valued as demonstrated by its participants, its practice, and its outcomes.

In a discussion of developing mathematical reasoning, Malloy states, "effective mathematics instruction of students with diverse cultural backgrounds challenges the notion that one culture or one method of reasoning dominates the learning process. In order for all children to become mathematically literate, mathematics educators must be poised to use varied instructional strategies that address the needs of the diverse populations of children who will live and become productive in the twenty-first century" (Stiff and Curcio 1999, 13).

In effective service learning, diversity is valued. Mathematics educators need to be poised to use varied instructional strategies that address the needs of the diverse populations of children in today's classrooms.

A meta-analysis of research on the learning of mathematics (Grouws and Cebulla 1999, 124) indicates that the use of small groups of students to work on tasks that deal with important mathematical concepts and ideas can increase student mathematics achievement. There is a richness and diversity of input of solution strategies, there is clarification of the problem itself, individual members can work on various parts of the solution, and evaluation of solutions is quite automatic; students check conclusions with one another. Conclusions are more valid because of the diversity of input. By the same token, focus on the task minimizes differentness of approach.

In effective service-learning, communciation and interaction with the community are promoted. The learning of mathematics is a social process; seeking and giving community input can serve as a performance measure of effectiveness of classroom efforts.

8. In effective service-learning, communication and interaction with the community are promoted and partnerships and collaboration are encouraged.

Children are constructors of their own mathematical knowledge (von Glasersfeld 1995); learning is a social process (Vygotsky 1978), and writing and talking are tools for reflecting thought as well as generating new thoughts (Murray 1968; Vygotsky 1978). Mathematics classrooms that engage students in formulating and testing hypotheses, discussing various approaches to problem-solving, keeping explorations open-ended so students feel free to contribute a diversity of responses, sharing personal mathematical connections so they develop mathematical ownership, and questioning solutions so students can learn to consider and defend their work (Whitin and Whitin 2000, 213–214), help students gain confidence in their mathematical abilities.

Certainly, taking those learnings one step further by seeking and giving community input can only strengthen such efforts. It can also serve as a performance measure of effectiveness of such classroom efforts. It is important that many classroom experiences precede actual community interaction so that confidence levels run high and tactics are many and varied.

In effective service-learning, students are prepared for all aspects of service work. Classrooms in which problem-solving discourse is usual build mathematics understanding and norms for social interaction.

9. In effective service-learning, students are prepared for all aspects of their service work. They possess a clear understanding of tasks and roles, as well as the skills and information required by the tasks; awareness of safety precautions; and knowledge about and sensitivity to the people with whom they will be working.

In a discussion of perspectives on mathematics education, Willoughby (2000, 9–10) analyzed that all people should have the following:

- A solid understanding of the significance and use of numbers
- Proficiency in the basic operations
- Ability to use thinking to solve problems (as opposed to tedious algorithms)
- Ability to decide which operations, arguments, or other mathematical thinking are appropriate to a real solution
- Ability to decide when approximations or estimates are appropriate
- Ability to use technology intelligently, including when to use technology
- Ability to collect, organize, and interpret data intelligently, to extrapolate or interpolate from the data, and to recognize unsound statistical procedures
- An understanding of probability concepts
- An understanding of the role of functions in modeling the real world
- A knowledge of the various geometric relationships
- A knowledge of and ability to use simple useful trigonometric information
- An intuitive understanding of the foundations of calculus

The ability to recognize situations in which mathematical thinking is likely to be helpful, to formulate problems in mathematical terms, and to interpret the results of mathematical analysis so that others can understand them, is useful in service-learning situations.

Classrooms in which problem-solving discourse is usual, where mathematical conjectures are posed and checked as a matter of course, and where solutions are explained and defended help students become in-depth thinkers. Not only do the classroom endeavors build mathematical understanding, but they also build norms for social interaction, problem investigation, and solution interpretation. With practice, these techniques would lend themselves well to broader venues.

In effective service learning, student reflection takes place. Well-taught mathematics carries students forward in knowing how to evaluate results.

10. In effective service learning, student reflection takes place before, during, and after service; uses multiple methods that encourage critical thinking; and is a central force in the design and fulfillment of curricular objectives.

When students become aware of themselves as learners, they can develop the ability to monitor their learning strategies and resources and can assess their readiness for particular performances (Bransford, Brown, and Cocking 1999, 55).

1) Metacognition can help students "analyze what they did and why . . . highlight the generalizable feature of the critical decisions and actions and focus on strategic levels rather than on specific solutions.
2) Building on previous experiences and identifying connections between the old learning and the new helps students increase transfer.
3) Students need to move pieces of knowledge to the conceptual level to increase understanding. "Conceptual understanding requires a higher-level, integrative thinking ability that needs to be taught systematically through all levels of schooling. Drawing patterns and connections between related facts, ideas, and examples leads to synthesizing information at a conceptual level" (Erickson 1998, 8). Critical content needs to correlate to disciplinary concepts and conceptual ideas.

If we can produce students who think critically, they will be able to handle not only academic tasks but life challenges. Well-taught mathematics carries students forward in knowing how to identify critical components of tasks, formulate solutions and execute them step-by-step, and evaluate results. These pieces lend themselves well to the broader curriculum and ultimately to the community.

In effective service learning, multiple methods are designed to validate students' service work. "Humans are motivated to develop competence and to solve problems."

—White, 1959, in Bransford, Brown, and Cocking 1999, 48

11. In effective service learning, multiple methods are designed to acknowledge, celebrate, and further validate students' service work.

"Learners of all ages are more motivated when they can see the usefulness of what they are learning and when they can use that information to do something that has an impact on others—especially their local community" (McCombs, 1996; Pintrich and Schunk, 1996). Social opportunities affect motivation.

"Although extrinsic rewards and punishments clearly affect behavior, people work hard for intrinsic reasons as well" (Bransford, Brown, and Cocking, 1999, p. 48). "Humans are motivated to develop competence and to solve problems" (White, 1959, in Bransford, Brown, and Cocking, 1999, p. 48).

And, does service learning work? A decade of research on service learning is yielding interesting results. Though much of the research used self-reports

Service learning provides an avenue for students to become active, positive contributors to society.

or information from surveys administered before and after a service experience, some used qualitative methods and case studies. The major findings of the research are (Billig 2000, 658–664):

1) Service learning has a positive effect on the personal development of public school youths.
2) Students who participate in service learning are less likely to engage in "risk" behaviors.
3) Service learning has a positive effect on students' interpersonal development and the ability to relate to culturally diverse groups.
4) Service learning helps develop students' sense of civic and social responsibility and their citizenship skills.
5) Service learning provides an avenue for students to become active, positive contributors to society.
6) Service learning helps students acquire academic skills and knowledge.
7) Students who participate in service learning are more engaged in their studies and more motivated to learn.
8) Service learning is associated with increased student attendance.
9) Service learning helps students to become more knowledgeable and realistic about careers.
10) Service learning results in greater mutual respect between teachers and students.
11) Service learning improves the overall school climate.
12) Engaging in service learning leads to discussions of teaching and learning and of the best ways for students to learn.
13) Service learning leads to more positive perceptions of schools and youths on the part of community members.

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